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2014 Smart Mater. Struct. 23 035013

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Magnetic actuation of a cylindrical microrobot using time-delay-estimation closed-loop control: modeling and experiments

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Received 1 July 2013, revised 18 December 2013
Accepted for publication 21 January 2014
Published 14 February 2014

Abstract
Accurate control systems are critical for safe and practical utilization of microrobots for in vivo operations. There exist uncertainties from the microrobot dynamics and nonlinearities from the magnetic driving force in the electromagnetic in vivo manipulation of microrobots, especially in low Reynolds number fluid flow. We describe the modeling and implementation of a closed-loop control system for a magnetically actuated microrobot based on time-delay estimation (TDE). The microrobot consisted of a cylindrical magnet, 0.5 mm in diameter and 1 mm in length, and the controller used optical sensing for position feedback. In addition, we describe an analytical model to determine the magnetic field components and the field gradients of a custom set of coils used to actuate the microrobot. Simulations were carried out to investigate the properties of the TDE control system, and it was subsequently tested experimentally, demonstrating that it provides robust control of the microrobot trajectory and does not exhibit chattering to follow step inputs.

Keywords: microrobotics control, electromagnetic steering, time delay estimation, magnetic field modeling

(Some figures may appear in colour only in the online journal)

1. Introduction

The use of micro- and nanorobots for delicate in vivo operations requires accurate and reliable closed-loop control. Applications for such robots include cancer therapies, such as hyperthermia therapy [1, 2] and brachytherapy [3]. In these therapies, a microrobot can be injected into the patient and travels to the location of the tumor. Once at the target destination, the microrobot locally heats or irradiates the surrounding tissue to destroy the cancer cells while limiting damage to healthy tissue. These machines can also be used for targeted drug delivery to locally dispense chemotherapy agents at the site of tumors [4]. Accurate trajectory control of such microrobots is also important in applications for diagnostic devices, such as in endoscopy [5, 6], where controlled motion of a tiny diagnostic tool can provide high-resolution imaging. In reaching a target location inside the human body, precise control over the robot path is necessary to minimize health risks during in vivo operations and to ensure that the target destination is reached.

The field of microrobotics has progressed considerably during the past decade. Several methodologies have been proposed and demonstrated for motile microrobots in viscous fluid environments where the Reynolds number ($Re$) is significantly less than unity. These microswimming robots are usually biomimetic, employing techniques inspired by...
microorganisms and bacteria, which use cilia and flagella to propel themselves [7–10]. However, a simpler method is to use a permanent magnet or a soft magnet in the fluid [11], manipulated using an external magnetic field gradient. The systems used to generate these magnetic fields include permanent magnets [12, 13]; Helmholtz coils, Maxwell coils, or a combination thereof [14–16]; magnetic resonance imaging (MRI) systems [17–19]; and customized sets of electromagnetic coils [20–23].

Such magnetic actuation systems have generally been used to visually control microrobots in an open-loop manner [21, 24]. For example, magnetic-field generation and imaging capabilities of MRI systems have been employed for untethered propulsion of magnetic micro-objects in a fluid [17–19]. However, to overcome the limited range of the generated magnetic field and inaccuracies in positioning [19], customized magnetic actuation systems consisting of several electromagnetic coils in optimized configurations have been developed [20, 21]. The use of electromagnetic coils provides the flexibility to control the direction and strength of magnetic fields by controlling the current in the coils. Finite element method (FEM) modeling has been used to establish relationships between the magnetic field in the coils and the applied current, and the results have been calibrated by comparison with experimentally measured data [21]. A point-dipole model is another method used to obtain the magnetic field of a coil; however, the model is valid only at large distances from the coil [25]. An analytical model that relates the magnetic field and magnetic field gradients to the current passing through the coil and is valid for all points outside the coil would be extremely useful for modeling these electromagnetic actuation systems. Such a model is advantageous because the computation cost is slight compared with FEM calculations, so it can be used to determine the input currents to the coils for a desired magnetic field quickly. In closed-loop control of microrobots, such an analytical model can be applied to find the input condition for the coils to give a desired force on the microrobot.

Closed-loop control of microrobots is more challenging because of the following considerations. For magnetic manipulation of microrobots, there are uncertainties in the following parameters: the dynamic properties of the robot, for example, the momentum and moment of inertia; the magnetic properties of the microrobot; the viscous and electromagnetic forces; the fluid dynamics; as well as nonlinearities in the magnetic driving force.

Unlike bacteria, artificial microswimmers are not naturally buoyant, and so it is important to prevent sinking by compensating for the weight of the microrobot in the closed-loop control system [26]. However, fabrication and measurement errors introduce uncertainties into the mass of the microrobot, on which the control law is based [26, 27]. Other sources of uncertainty in microrobot dynamics include the hydrodynamic force. The drag force exerted from fluid on an object moving through that fluid is linearly dependent on the object velocity at low Reynolds number flows ($Re \ll 1$). The drag coefficient, which relates the drag force and velocity, varies for different microrobot shapes and orientations. The drag coefficient has been formulated only for special body shapes, and it is difficult to make measurements of the drag forces of objects with arbitrary shapes when $Re$ is small. The viscosity of the fluid varies with temperature, which results in unpredictable variation of the drag force. Errors in the computation of the drag coefficient lead to uncertainties in the microrobot dynamics, which can result in severe control issues. In some applications, there are other forces that are difficult to measure, such as contact, van der Waals, and electrostatic forces, as well as wall effects, all of which contribute to the overall uncertainty in the dynamics [28]. Modeling these forces is associated with various errors, which can be improved using empirical correction factors [29].

The proportional–integral–derivative (PID) controller is usually used for closed-loop control of microrobots. Since PID control is a linear method, which cannot take into account the uncertainties and nonlinearities of the microrobot motion, other methods have been proposed [27, 30, 31]. However, these control algorithms, such as H-infinity control, require accurate modeling of the system and are highly mathematical, making the optimization complex and time consuming [27].

In this paper, we propose a closed-loop control system for three-dimensional translational motion of a microrobot based on time-delay estimation (TDE). The microrobot is driven by a customized set of electromagnetic coils. We analytically model the magnetic fields and spatial field gradients from the coils, and compare the numerical results with experimental data. The dynamics of the microrobot is modeled, and the formulation of the TDE control is described. The dynamic model contains uncertainties and nonlinearities; however, the TDE controller performance can be designed based on second-order error dynamics to accommodate these factors. An optical tracking system is used for position feedback to the controller. To account for the inherent uncertainty in the dynamic parameters of artificial microswimmers, the TDE control method employs an effective and efficient estimation of the unknown forces acting on the swimmers. For this reason, this method is appropriate to control motion with a large number of degrees of freedom. Furthermore, our method exhibits a rapid response, resulting in motion that is free from chattering, which has previously been shown to be an issue in microrobot systems [27].

In addition to implementing a TDE control method, we report analytical expressions to model the magnetic field components and gradients for a set of electromagnetic coils. This analytical model can be used to find the set of input coil currents for a given desired force on the microrobot. The rest of this paper is organized as follows. Section 2 describes the modeling of the magnetic actuation system. The microrobot equations of motion are given in section 3 and the TDE controller is described in section 4. The experimental procedure and results are presented in section 5, and the paper is summarized in section 6.

2. Magnetic actuation modeling
2.1. Magnetic field of a set of electromagnetic coils
We developed an analytical model that takes the currents in a set of electromagnetic coils as inputs, and calculates
the magnetic field and magnetic field gradients within the workspace. The dimensions and configuration of the set of coils are provided as inputs to the model. The electromagnets are placed under the workspace. Using our model, it is possible to calculate the magnetic field at any point outside the coils.

In the vicinity of an electromagnetic air-core coil, the magnetic flux density is proportional to the current passing through the coil. The magnetic flux density is increased when a ferromagnetic core is used with the coil wrapping around the core. In this case, the generated magnetic field depends on the relative permeability of the metallic core, $\mu_r$, and the current in the coil. This results in a nonlinear relationship between the magnetic flux density and the current. However, for long thin coils, we can neglect the dependence of the permeability of the core on the current, and assume that the magnetic flux density at point $P$ varies linearly with the current, i.e.,

$$B(P) = B(P)I.$$  \hfill (1)

where $I$ denotes the current in the coil and $B(P)$ stands for a $3 \times 1$ matrix, which is a function of position and can be evaluated at any point outside the coil. A coil is composed of $N$ current loops; we determine the magnetic field of each current loop and superimpose the contribution of each loop to obtain the total flux density and gradient.

Since $\nabla \cdot B = 0$ everywhere for a loop of wire with current $I$, as shown in figure 1, the magnetic flux density can be written as $B(P) = \nabla \times A(P)$, where $A(P)$ denotes the vector potential. $A(P)$ is defined by the Biot–Savart law for an element of the form

$$A(P) = \frac{\mu_0 I}{4\pi} \oint_C \frac{dl}{r_e},$$  \hfill (2)

where $\mu_0$ denotes the permeability of free space and $r_e$ a position vector connecting the element to point $P$. The integration is taken over the entire loop contour $C$. The integral of equation (2) is converted into a definite integral as follows [25]:

$$A(P) = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{a \cos \psi}{r_e} d\psi.$$  \hfill (3)

One can numerically evaluate the integral in equation (3) by using complete elliptic integrals of the first $(K)$ and second $(E)$ types. Analytical expressions are given for the magnetic field components at any point $P(x, y, z)$ outside the loop as follows [32]:

$$B_x = \frac{\mu_0 I}{\pi} \frac{xy}{2a^2 \beta \rho^2} [\{a^2 + r^2\} E(k^2) - a^2 K(k^2)]$$  \hfill (4)

$$B_y = \frac{\mu_0 I}{\pi} \frac{yz}{2a^2 \beta \rho^2} [\{a^2 + r^2\} E(k^2) - a^2 K(k^2)]$$  \hfill (5)

$$B_z = \frac{\mu_0 I}{\pi} \frac{1}{2a^2 \beta} [\{a^2 - r^2\} E(k^2) + a^2 K(k^2)]$$  \hfill (6)

where

$$\rho = \sqrt{x^2 + y^2}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\alpha = \sqrt{a^2 + r^2 - 2a\rho}$$

$$\beta = \sqrt{a^2 + r^2 + 2a\rho}$$

$$\kappa^2 = 1 - \frac{\alpha^2}{\beta^2}$$

and

$$\gamma = x^2 - y^2.$$  \hfill (12)

The spatial derivatives of the magnetic flux density are obtained from [32]:

$$\frac{\partial B_x}{\partial x} = \frac{\mu_0 I}{\pi} \frac{z}{2a^2 \beta \rho^4} \left\{ \left[ a^4 (\alpha - \gamma (3z^2 + a^2) + \rho^2 (8x^2 - y^2)) \right. \right.$$

$$\left. - a^2 (\rho^4 (5x^2 + y^2) - 2\rho^2 z^2 (2x^2 + y^2) + 3z^4 \gamma) \right.$$

$$- r^4 (2x^4 + \gamma (y^2 + z^2)) \right\} E(k^2)$$

$$+ a^2 [(a^2 (\gamma (a^2 + 2z^2) - 2\rho^2 (3x^2 - 2y^2))]$$

$$+ r^2 (2x^4 + \gamma (y^2 + z^2)) K (k^2) \right\}$$

$$\frac{\partial B_x}{\partial y} = \frac{\mu_0 I}{\pi} \frac{xyz}{2a^2 \beta \rho^4} \left\{ \left[ 3a^4 (3\rho^2 - 2z^2) - r^4 (2r^2 + \rho^2) \right. \right.$$

$$\left. - 2a^6 - 2a^4 (2\rho^4 - \rho^2 z^2 + 3z^4) \right\} E(k^2)$$

$$+ a^2 \left[ r^2 (2r^2 + \rho^2) \right.$$$$\left. - a^2 (5\rho^2 - 4z^2) + 2a^4 \right] K (k^2) \right\}$$

$$\frac{\partial B_x}{\partial z} = \frac{\mu_0 I}{\pi} \frac{x}{2a^2 \beta \rho^4} \left\{ \left[ (\rho^2 - a^2) (2\rho^2 + a^2) \right. \right.$$

$$\left. + 2z^2 (a^4 - 6a^2 \rho^2 + \rho^4) + z^4 (2a^4 + \rho^2) \right\} E(k^2)$$

$$- a^2 (\rho^2 - a^2) ^2 + z^2 (\rho^2 + a^2) \right\} K (k^2) \right\}$$

$$\frac{\partial B_y}{\partial x} = \frac{y}{x} \frac{\partial B_x}{\partial z}$$

$$\frac{\partial B_y}{\partial x} = \frac{\partial B_z}{\partial z}$$

$$\frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial z}$$

$$\frac{\partial B_z}{\partial z} = \frac{\mu_0 I}{\pi} \frac{z}{2a^2 \beta \rho^4} \left\{ \left[ 6a^4 (\rho^2 - z^2) - 7a^4 + r^4 \right] E(k^2) \right. $$

$$+ a^2 (a^2 - r^2) K (k^2) \right\}.$$  \hfill (21)
The permeability of coils with a ferromagnetic core was $\mu_0\mu_r$. The components of the magnetic field could then be transformed back to the global coordinate system where they were superimposed, i.e.,

$$B(P) = \sum_{i=1}^{8} B_i(P)I_i$$

(22)

to obtain the global magnetic field.

To investigate the validity of the model, we provided a set of coil currents to the model as $I = [-1.54, 0.94, 1.49, 1.07, 1.3, 1.18, 1.33, 1.13, 1.49]^T$. Figure 3 shows the calculated magnetic field strength and components for the workspace in the $z = 0$ plane. The magnetic field strength varied in the range 3–6 mT for the $z = 0$ plane and it was 5 mT at the workspace center. The measured magnetic-field vector mapping for the $xz$ and $yz$ planes passing through the workspace center is shown in figure 4. The magnetic field was homogeneously directed along the $z$-axis around the workspace center, but it deviated from the desired direction towards the edges of workspace.

Table 1 compares the magnetic field evaluated by the model with measured data by a three-axis hall magnetometer (THM1176, Metrolab Instruments SA, Switzerland) at certain points within the workspace. The model showed good agreement with the measured data, especially around the workspace center.

### 2.2. Magnetic torque and force

The orientation and position of a magnetic microrobot can be controlled by the applied magnetic torque and force. The torque that a magnetic field applies to the object is described by

$$T_m = m \times B,$$

(23)

which aligns the microrobot magnetic moment vector $m$ with the magnetic flux density vector $B$. For translational manipulation, a spatial gradient of the magnetic field exerts a force on the magnetic microrobot. The magnetic force is expressed as

$$F_m = (m \cdot \nabla)B.$$

(24)

If the condition $\nabla \times B = 0$ is satisfied, we can rewrite equation (24) in a matrix form as

$$F_m = \begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix} \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} m^T B_x \\ m^T B_y \\ m^T B_z \end{bmatrix} \mathbf{I},$$

(25)

where the vector $\mathbf{I}$ describes the currents in the coils. In equation (25), all spatial derivatives of the magnetic flux density are evaluated at the location of the microrobot. Combining equations (22) and (25) yields

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{F}_m \end{bmatrix} = \begin{bmatrix} \mathbf{B}(P) \\ m^T B_x(P) \\ m^T B_y(P) \\ m^T B_z(P) \end{bmatrix} \mathbf{I} = \mathbf{C}(m, P)\mathbf{I},$$

(26)
Figure 3. Mapping of the (a) magnetic field strength, (b) magnetic field $x$-components, (c) magnetic field $y$-components and (d) magnetic field $z$-components in the $z = 0$ plane of the workspace for a set of coils current of $I = [-1.54, 0.94, 1.49, 1.07, 1.3, 1.18, 1.33, 1.13, 1.49]^T$.

Figure 4. Mapping of the magnetic field components for a set of coils current of $I = [-1.54, 0.94, 1.49, 1.07, 1.3, 1.18, 1.33, 1.13, 1.49]^T$ in the (a) $xz$ plane and (b) $yz$ plane, both passing through the workspace center.

Table 1. Magnetic field evaluated by the model and measurement for a set of coils current of $I = [-1.54, 0.94, 1.49, 1.07, 1.3, 1.18, 1.33, 1.13, 1.49]^T$.

| $[x, y, z]$ | Model (mT) | Measurement (mT) | $|B_{\text{model}}|/|B_{\text{measurement}}|$ |
|------------|------------|-----------------|-----------------|
| [0, 0, 0]  | [-0.07, -0.01, 5.15] | [0.34, -0.07, 5.38] | 0.96 |
| [-5, 0, 0] | [-0.18, -0.13, 3.11] | [-0.47, -0.14, 4.24] | 0.73 |
| [5, 0, 0]  | [0.68, 0.79, 6.08] | [0.44, 0.22, 5.73] | 1.07 |
| [0, -5, 0] | [-1.22, -1.03, 4.68] | [-0.53, -0.03, 5.82] | 0.85 |
| [0, 5, 0]  | [1.07, 1.01, 4.68] | [-0.32, 0.03, 5.48] | 0.89 |
The matrix \( C \) is elaborated using the analytical model described in section 2.1. For a desired magnetic field and magnetic force, one can solve equation (26) using the pseudoinverse of \( C \) to obtain the required currents of the coils, i.e.,

\[
I = C^\dagger (m, P) \begin{bmatrix} B \end{bmatrix}_{\text{desired}} .
\]

(27)

Once the desired force, magnetic moment, and position of microrobot are known, the actuation system specifies the current in the coils based on equation (27).

3. Microrobot dynamics

A magnetic cylinder can be manipulated by the force due to the gradient of a magnetic field in the three translational degrees of freedom. A cylindrical magnetic microrobot is shown in figure 5 along with the coordinate system. The equation of linear motion for the microrobot moving in the fluid is obtained by using conservation of linear momentum as

\[
F_m + F_{\text{drag}} + F_g + F_b = ma,
\]

(28)

where \( F_m \) denotes the applied magnetic force, \( F_{\text{drag}} \) the hydrodynamic drag exerted by the fluid on the magnetic cylinder, \( F_g \) the gravitational force, and \( F_b \) the buoyancy force. \( m \) and \( a \) stand for mass and acceleration of the cylinder.

The drag force for a moving object in a low \( Re \) number flow varies linearly with velocity and is in the direction opposite to the velocity. For an asymmetric body in creeping flow, the drag force depends on the orientation. When the cylinder is aligned with the z-axis, the drag force in the z direction, \( F_{\text{drag},z} \), can be approximated by the drag on a sphere, which can be expressed as

\[
F_{\text{drag},z} = -3\pi \mu D u_z,
\]

(29)

where \( D \) denotes the diameter of the cylinder, \( \mu \) dynamic viscosity of the fluid, and \( u_z \) the z-axis component of the object velocity. However, the drag force components in the x- and y-directions differ from that in equation (29). The analytical expression for the drag force in the direction perpendicular to the cylinder axis is given by [33]

\[
F_{\text{drag},x} = -\frac{4\pi \mu L}{\ln\left(\frac{7\mu D}{\rho L}\right)} u_x,
\]

(30)

where \( F_{\text{drag},x} \) and \( u_x \) denote the drag force and velocity in x direction, respectively, \( L \) the cylinder length, \( Re = \frac{\rho u L}{\mu} \) the \( Re \) number, and \( \rho \) the density of the fluid. The relationship between the drag force and the velocity in the y direction is similar to equation (30). Combining equations (29) and (30), the total drag force is obtained as

\[
\mathbf{F}_{\text{drag}} = -\begin{bmatrix} \frac{4\pi \mu L}{\ln\left(\frac{7\mu D}{\rho L}\right)} & 0 & 0 \\ 0 & \frac{4\pi \mu L}{\ln\left(\frac{7\mu D}{\rho L}\right)} & 0 \\ 0 & 0 & 3\pi \mu D \end{bmatrix} \mathbf{u}.
\]

(31)

4. Control by time-delay estimation

We used the TDE method to control the trajectory of the microrobot to a target along a desired path. TDE uses the previous time-step information of the system variables to estimate the current values and has been applied to several robotic systems [34–36]. Systems with nonlinearities and uncertainties can be properly controlled with a straightforward gain design and the implementation of TDE is relatively uncomplicated [37].

To obtain the control law of our system based on TDE, we rewrite equation (28) using the position vector of the microrobot, \( \mathbf{P} \)

\[
F_m = m\ddot{\mathbf{P}} - (F_{\text{drag}} + F_g + F_b),
\]

(32)

where \( \dot{\mathbf{P}} \) is the second time derivative of the microrobot position. Multiplying the acceleration by a constant \( m_0 \) to have \( m_0\ddot{\mathbf{P}} \), which then is added to and subtracted from the right-hand side of equation (32), we have

\[
F_m = m_0\ddot{\mathbf{P}} + \mathbf{H},
\]

(33)

where \( \mathbf{H} = (m - m_0)\ddot{\mathbf{P}} - (F_{\text{drag}} + F_g + F_b) \) includes all terms that contain uncertainty. Contact forces, van der Waals forces, and electrostatic forces are included in this parameter. Note that equation (33) is a different expression of equation (32).

Based on equation (33), the input force for control, \( \mathbf{F}_c \), is constructed as

\[
\mathbf{F}_c = m_0\mathbf{V} + \dot{\mathbf{H}},
\]

(34)

where \( \dot{\mathbf{H}} \) denotes an estimate of \( \mathbf{H} \), and \( \mathbf{V} \) is given by

\[
\mathbf{V} = (\mathbf{P}_d + \mathbf{K}_d (\mathbf{P}_d - \dot{\mathbf{P}})) + \mathbf{K}_p (\mathbf{P}_d - \mathbf{P}),
\]

(35)

where \( \mathbf{P}_d \) denotes the desired position vector; diagonal matrices \( \mathbf{K}_p \) and \( \mathbf{K}_v \) stand for proportional and derivative gains, respectively, the diagonal entries of which are \( K_{px}, K_{py}, K_{pz} \) and \( K_{vx}, K_{vy}, K_{vz} \), respectively.
Provided that $\hat{H}$ is sufficiently accurate, application of the input force $F_i$ to the linear motion of the microrobot, that is substitution of equations (34) and (35) into equation (33), leads to the following second-order closed-loop error dynamics:

$$\ddot{e} + K_p \dot{e} + K_v e \approx 0,$$  \hspace{1cm} (36)

where $e$ denotes the position error, defined as $e = P_0 - P$. The gains, $K_p$ and $K_v$, can be easily designed so that the error dynamics in (36) may exhibit a desired response. These gains, being diagonal matrices, can be designed individually in three directions to cope with the different dynamics of the microrobot in the three directions.

The main difficulty for the control system is how to select $\hat{H}$ that accurately estimates $H$. A powerful solution is using TDE. Provided that the terms included in $H$ are continuous or at least piecewise continuous, $H$ can be estimated based on the previous time-delayed value of the system variables. By virtue of the continuity of $H$, the following holds

$$H(t - \delta) \rightarrow H(t), \hspace{1cm} \text{as } \delta \rightarrow 0,$$

where $\delta$ denotes the time delay. Hence, we can use $H(t - \delta)$ as a good estimate of $H(t)$; namely $\hat{H}(t) = H(t - \delta)$. Use of the time-delayed information as an estimate is the key idea of TDE. Then from equation (33) one obtains

$$\hat{H} = H(t - \delta) = F_m(t - \delta) - m_0 \ddot{P}(t - \delta).$$  \hspace{1cm} (37)

However in practice, $\delta$ is a finite value which is induced by the sampling frequency of the digital control system. The total control law can be obtained by combining equations (34), (35) and (37) as

$$F_m(t) = m_0 (\ddot{P}_d + K_v \dot{e} + K_p e) + F_m(t - \delta) - m_0 \ddot{P}(t - \delta),$$  \hspace{1cm} (38)

where $m_0$ is a parameter used for tuning the controller performance. Note that in (38) the control law requires position information, $P(t)$, and its successive time derivatives up to $\ddot{P}(t)$, which are made available by numerical differentiation. Figure 6 shows a schematic diagram describing the operation of the controller.

It is noteworthy that the control law does not require any information about the system model except for the order of the system; the other terms, $P_d$, $m_0$, $K_p$, $K_v$, are either a design variable or design parameters.

5. Results

The dynamics of the microrobot and the proposed controller were simulated using MATLAB®/Simulink® (MathWorks, Inc.). The microrobot was modeled as a cylindrical NdFeB permanent magnet, 0.5 mm in diameter and 1 mm in length. The density was 7450 kg m$^{-3}$, the magnetic moment was 0.00027 A m$^2$, and the mass was 1.46 mg. The surrounding fluid was silicone oil, with a dynamic viscosity of 1 Pa s. The drag force was modeled using equation (31), where the $Re = 4.85 \times 10^{-4}$ was used to describe a mean microrobot velocity of 1 mm s$^{-1}$. The drag coefficients were $d_c = d_s = 1.3 \times 10^{-3}$ and $d_t = 4.71 \times 10^{-5}$. The time interval between samplings was $\delta = 66.67$ ms, which was considered for the minimum available frequency of camera.

The value of the operating gains, $K_p$, $K_v$, as well as $m_0$ should be chosen independently for the three directions. The design procedure for the controller gains is straightforward. The desired error dynamics was determined by selecting a natural frequency, $\omega_n$, and a damping ratio, $\zeta$. Based on equation (36), the gains were $K_p = \omega_n^2$ and $K_v = 2 \zeta \omega$. The value of $m_0$ depends on the sampling frequency, which is determined by the hardware and should be tuned to a given implementation. One can start with $m_0$ close to or less than the microrobot mass and vary this based on the controller performance (further details on this can be found in [35]).

The controller performance was investigated experimentally using a magnetic actuation system, two cameras (GRAS-50S5C-C, Point Grey CCD Camera, Canada), with lenses (VZM 600i, Edmund Optics, USA) and camera holders as shown in figure 7. A cubic plastic container held the silicone oil, which had a dynamic viscosity of 1 Pa s (KF96-1000CS, Shin-Etsu Silicone, Korea). In the absence of a magnetic field, the microrobot sank in the fluid due to the difference in density between the NdFeB and the silicone oil.

The container was placed on the magnetic actuation system (as shown by the white box in figure 7). The position of the microrobot was determined using the two cameras: one providing a top view and the other providing a side view that was inclined at $17^\circ$ relative to the horizontal plane. The moving object was detected and tracked using background subtraction and its position was estimated after filtering out the motion.
The experiments had a frequency of 15 Hz and a resolution
noises using morphological operations. The cameras used in
which has eight electromagnetic coils, and two cameras, lenses, and
camera holders.

![Magnetic actuation system](image)

**Figure 7.** Experimental setup used to characterize the TDE control
of the microrobot. The setup includes a magnetic actuation system,
which has eight electromagnetic coils, and two cameras, lenses, and
camera holders.

5.1. Single-step input

We evaluated the performance of the controller for single-step
inputs in three dimensions. A single target point was specified
as a step input in the experiment. The microrobot was aligned
with the $z$-axis. Figure 8 shows the experimental results of
the microrobot position following the step inputs of 0.4 mm,
0.3 mm, and 0.9 mm in the $x$, $y$, and $z$ directions, respectively.
The required control input forces for steering the microrobot
to the desired target are also shown in figure 8. All results in
figure 8 are given for three different gains, each of which
produced different error dynamics. The controller performance
characteristics are summarized in table 2, and compared with
simulations using identical gains in the three directions, specifically $K_p = 100$ and $K_v = 120$, which corresponds to error
dynamics of $\omega_n = 10$ and $\zeta = 6$. The tuning parameter $m_0 = 3 \times 10^{-7}$ was used in both the experiments and simulations.

<table>
<thead>
<tr>
<th>Performance parameter</th>
<th>Simulation</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$</td>
<td>$y$</td>
</tr>
<tr>
<td>95% Rise time (s)</td>
<td>3.60</td>
<td>3.60</td>
</tr>
<tr>
<td>Overshoot (%)</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>±5% Settling time (s)</td>
<td>3.60</td>
<td>3.60</td>
</tr>
</tbody>
</table>

To prevent overshoot in the controller response during the
experiments, we increased the damping ratio in the desired
error dynamics. The gains were $K_{p_x} = K_{p_y} = 66.7$ and $K_{p_z} = 73.3$; $K_v$ was maintained at 120 for all three directions which
implies a damping ratio of $\zeta_x = \zeta_y = 7.35$ and $\zeta_z = 7.01$. This resulted in a rise time of 0.82 s, 0.76 s, and 2.08 s, a settling
time of 3.24 s, 4.21 s, and 4.36 s, and an overshoot of 51.1%,
48.1%, and 7.6% in the $x$, $y$, and $z$ directions, respectively. Further increasing the damping ratio to $\zeta_x = \zeta_y = 18.97$ and
$\zeta_z = 7.75$ reduced the overshoot of the step response to 11.2%,
16.5%, and 2.5% using $K_p$ gains of 10, 10, and 60 in the $x$, $y$, and $z$ directions, respectively, while maintaining $K_v = 120$.

The controller required a relatively high damping ratio to
overcome the overshoot of the step response. This occurred for two reasons. First, the controller performance depends on the time delay. Increasing the sampling rate (by increasing the camera speed) will reduce the time delay and results in a less
overshoot and faster controller response. Second, the inherent
nonlinearities of the magnetic actuation were not included
in the simulation. These nonlinearities associated with the
unmodeled forces, including stiction, which is present at the
initial position when the microrobot is in contact with the
container surface, are sources of the overshoots observed in
the experiment; however, they do not exist in the simulations.

5.2. Successive step inputs

We considered a path consisting of four points to be reached
by the microrobot successively. The targets were corners of
a rectangle in the $z = 0$ plane with a length of 3 mm and width
of 2 mm. The initial position of the microrobot was at the
center of the rectangle, and the orientation of the microrobot
was aligned with the $z$-axis. Figures 9 and 10 show the frames
of controlled manipulation of the microrobot from the top and
side views. A target was considered reached when the center
of the microrobot coincided with any point within a locus of
100 $\mu$m from the target position. The experiment was carried
out with gains of $K_{p_x} = K_{p_y} = 66.7$ and $K_{p_z} = 73.3$ for the
three directions, with $K_v = 120$.

Since the distance between the target points was larger
than the previous experiment, the overshoot was larger and
the settling time was longer. However, the controller did
not show any chattering around the targets. Furthermore, we
considered the microrobot weight as an unknown in finding
the control law. Figure 10 shows that the controller was able to
compensate for the microrobot weight and maintained a path
with a constant $z$-position. The total time for the microrobot
to traverse this path was 21.66 s.
5.3. Arbitrary path

We specified a circular path in the $xy$ plane defined by 40 points, with a radius of 1 mm. Again, the target zone around the points was such that the center of the microrobot came within a locus of 100 $\mu$m from the goal. The initial position of the microrobot was at the center of the circle. Figure 11 shows the desired and actual trajectories of the microrobot. This experiment was carried out with a gain design of $K_p = 100$ and $K_v = 120$ for the three directions. The total time was 49 s. The microrobot was aligned with the $z$-axis at all times. In following the desired trajectory, the reduced damping decreased the control time since the successive targets were close together, but it did not significantly affect the accuracy of the path.

5.4. Comparison with H-infinity

The TDE controller did not exhibit chattering following the step inputs and responded faster than using H-infinity control [27]. The advantages of a TDE control over H-infinity control or other model-based control algorithms for microscale applications include the following.

- In the TDE method, hydrodynamic, contact, electrostatic, and other forces present at microscales that are difficult to model or measure can be considered as unknowns in the control process. However, the accuracy of H-infinity depends on how well the system is modeled, since the H-infinity controller is only optimized for a given problem.
- TDE control has a more straightforward design procedure and is relatively simple to implement compared to H-infinity control [38].

6. Conclusions

A TDE controller for a magnetically actuated swimming microrobot in low-$Re$ fluid flow has been described. The dynamics of the microrobot were analyzed, and the synthesis of a control system has been provided. Simulations and
Figure 9. Top view of the controlled motion of the magnetic microrobot aligned with the $z$-axis. The target path was four points describing a rectangle in the $z = 0$ plane.

Figure 10. Side view of the magnetic microrobot when following the trajectory shown in figure 9 at the corresponding time frames. The controller was able to compensate for the weight of the microrobot, maintaining the microrobot at $z = 0$.

experiments were performed to determine the performance characteristics of the control system. An analytical model was described to determine the magnetic field and spatial gradients generated by a set of electromagnetic coils. The model was validated experimentally by measuring the magnetic field at certain points within the workspace.

The analytical model was able to predict the magnetic field and the gradient of the field with sufficient accuracy to develop a control system. The magnitude of the magnetic field evaluated by the model at the center of workspace was in agreement with the measured data to within $\pm 4\%$, and the orientation of the magnetic field was in agreement to within $2.5^\circ$. 
This analytical model was used in the design of a custom magnetic actuation system for micromanipulation and controlling the dynamics of a microrobot. The response to a step input, as well as to successive step inputs and a desired path demonstrated the performance of the control system. The microrobot was able to follow a step input with a rise time of 0.8 s, 0.72 s, and 1.74 s, an overshoot of 63.1%, 66.7%, and 18.0%, and a settling time of 3.04 s, 3.17 s, and 4.2 s in the $x$, $y$, and $z$ directions, respectively. The overshoot was reduced to 11.2%, 16.5%, and 2.5% by increasing the damping ratio of the error dynamics to $\zeta_x = \zeta_y = 18.97$ and $\zeta_z = 7.75$. The performance of the control system can be further improved by using a faster sampling rate for the feedback.

The TDE control system is simple to implement and does not require a detailed description of the system dynamics. The results described here demonstrate that nonlinearities and uncertainties can be compensated by using a straightforward and robust control method. The controller required input forces that are within the available range of custom-made magnetic actuation systems, and we have demonstrated the ability to accurately follow geometrical paths.

Acknowledgments

This work was supported by the DGIST R&D Program of the Ministry of Education, Science, and Technology of Korea (11-BD-0402 and 13-BD-0403) and DGIST MIREBraiN Project.

References


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